MATH 502 HOMEWORK 1

Due Monday, September 16.

Problem 1. *Let $\mathcal{L}$ be a language with relation symbols $\mathcal{R}$, function symbols $\mathcal{F}$ and constant symbols $\mathcal{C}$. Let $T_0 := C \cup \{v_1, v_2, ...\}$ be the variables and constants of the language. Given T_k, let $T_{k+1} = \{f(t_1, ...t_{n_f}) \mid f \in \mathcal{F}, t_1, ..., t_{n_f} \in T_k\}$. Set $T = \bigcup_k T_k$. Prove that T is the set of all terms.*

Problem 2. *Exercise 2.19. Then use this to do the Q2- inductive step of the proof of Soundness.*

Problem 3. *Exercise 3.7*

Problem 4. *Let M be an $\mathcal{L}$-structure and let $\sigma : V \to M$ be an assignment.*

a) Suppose that $M \models_\sigma \exists v_1 \forall v_2 \phi$. Prove that $M \models_\sigma \forall v_2 \exists v_1 \phi$. Give an example showing that the converse is false.

b) Give an example of $\mathcal{L}, M, \sigma, \phi$ and ψ, such that $M \models_\sigma (\exists v_1 \phi) \wedge \psi$, but $M \not\models_\sigma \exists v_1 (\phi \wedge \psi)$.

Problem 5. *Let $\mathcal{L} = \{E\}$, where E is a binary relation symbol. Define a theory T, such that $M \models T$ if and only if E^M is an equivalence relation and all equivalence classes are infinite.*